Numbering Systems and Comp...

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Numbering Systems and Computer Codes
Prepared by The Computer Information Systems Department

Decimal Numbering Systems:
The decimal numbering system is a base 10 numbering system (this means there are 10 digits we can use - these digits are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). When we talk about a number, we understand what the number is because of its face value and its positional value. Thus, the digit 5 has a different meaning when it is in the tens position than when it is in the ones position (i.e. when it is in the tens position, we express it as 50 and when it is in the ones position, we express it as 5). In this case, the face value of the digit is 5 and the positional value of a number is based on the position it occupies. In decimal, the positional value of a number is based on the powers of 10 (remember, we are in base 10):

5	7	2	4	Face value
10^3	10^2	10^1	10^0	Positional Value (powers of 10)
1000	100	10	1	Resolved positional value

To figure out the value of 5724, we do the following:

$5 \times 10^3 = 5 \times 1000 = 5000$
$7 \times 10^2 = 7 \times 100 = 700$
$2 \times 10^1 = 2 \times 10 = 20$
$4 \times 10^0 = 4 \times 1 = 4$
5724

Binary Numbering systems:
The binary numbering system works much the same way as the decimal numbering system except that now we are in base 2 so we only have 2 digits (0, 1). The value of the number is still determined by the face value times the positional value, but since we are in base 2, the positional values are the powers of 2. Since the face values can only be 0 or 1, this means that the 0 or 1 is multiplied by the positional place in which it is found.
Example: binary number 1011011

1	0	1	1	0	1	1	Face value
2^6	2^5	2^4	2^3	2^2	2^1	2^0	Positional value

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Binary Base 2 Digits are 0,1

$$\begin{array}{r} 1 \\ +\ 1 \\ \hline 10 \\ +\ 1 \\ \hline 11 \\ +\ 1 \\ \hline 100 \\ +\ 1 \\ \hline 101 \\ +\ 1 \\ \hline 110 \end{array}$$

1 max

$$\begin{array}{r} 110 \\ +\ 1 \\ \hline 111 \\ +\ 1 \\ \hline 1000 \\ +\ 1 \\ \hline 1001 \end{array}$$

Dec

$$\begin{array}{r} 9 \\ +\ 1 \\ \hline 10 \end{array}$$

9 max

This means that when we count, we get the following:

Decimal	Binary
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000

$$\begin{array}{r} 1 \\ +1 \\ \hline 10 \\ +1 \\ \hline 11 \\ +1 \\ \hline 100 \end{array}$$

Hexadecimal Numbering System:

The next numbering system is the hexadecimal numbering system. This is the base 16 numbering system, therefore there are 16 digits (0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F). The letter A carries the same value as decimal 10, the letter B carries the same value as decimal 11, the letter C carries the

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Example: binary number 1011011

1	0	1	1	0	1	1	Face value
2^6	2^5	2^4	2^3	2^2	2^1	2^0	Positional value
64	32	16	8	4	2	1	Resolved positional value

$10110_2 = \underline{22}_{10}$

16 8 4 2 1

The positional values are first shown in the powers of 2 and then as the resolved number - in other words, 2 to the 6th is equal to 64.

Converting binary to decimal:
In the previous example to find the decimal equivalent for the number 1011011, we do the following:

$1 \times 2^6 = 1 \times 64 =$	64
$0 \times 2^5 = 0 \times 32 =$	0
$1 \times 2^4 = 1 \times 16 =$	16
$1 \times 2^3 = 1 \times 8 =$	8
$0 \times 2^2 = 0 \times 4 =$	0
$1 \times 2^1 = 1 \times 2 =$	2
$1 \times 2^0 = 1 \times 1 =$	1
	91

Decimal

face value → 1 0 1 1 0

resolved positional value — 16 8 4 2 1

$$\begin{array}{r} 16 \\ +4 \\ +2 \\ \hline 22 \end{array}$$

Converting decimal to binary:
Before doing this it is important that we review the decimal equivalent for the frequently used powers of 2:

$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
$2^6 = 64$	$2^7 = 128$	$2^8 = 256$	$2^9 = 512$	$2^{10} = 1024$	etc.

$100111_2 = \underline{39}_{10}$

1	0	0	1	1	1
32	16	8	4	2	1
2^5	2^4	2^3	2^2	2^1	2^0

$$\begin{array}{r} 32 \\ 4 \\ 2 \\ 1 \\ \hline 39 \end{array}$$

$$1010101_2 = \underline{\quad 85 \quad}_{10}$$

64 32 16 8 4 2 1

64
16
4
1

85

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Search

Converting decimal to binary:
Before doing this it is important that we review the decimal equivalent for the frequently used powers of 2:

$1011011_2 = 91_{10}$

$2^0 = 1$	$2^1 = 2$	$2^2 = 4$	$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
$2^6 = 64$	$2^7 = 128$	$2^8 = 256$	$2^9 = 512$	$2^{10} = 1024$	etc.

To convert 91 from decimal to binary, you can follow the following steps:

1. Look at 91 and see what power of 2 can be taken from it. The highest power that can be subtracted is 2 to the 6th which is 64. Therefore we put a 1 in the 2 to the 6th position. Then we subtract: 91 - 64 = 27

	1	0	1	1	0	1	1
2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
128	64	32	16	8	4	2	1

$$\begin{array}{r} 91 \\ -64 \\ \hline 27 \\ -16 \\ \hline 11 \\ -8 \\ \hline 3 \\ -2 \\ \hline 1 \end{array} \quad 1-1=0$$

2. Now we look at what is left, 27 and see if the next power of 2 (moving to the right) which is 2 to the 5th or 32 can be subtracted from 27. It can't, therefore we didn't use the 2 to the 5th position so we put a 0 in the 2 to the 5th position. Since we didn't use the 32, there is no subtraction.

1	0	___	___	___	___	___
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

3. Now we check to see if the next power of 2 (moving to the right) which is 2 to the 4th with the value of 16 can be subtracted from 27. It can, therefore we put a 1 in the 2 to the 4th position. Then we subtract: 27 - 16 = 11.

1	0	1	___	___	___	___
2^6	2^5	2^4	2^3	2^2	2^1	2^0
64	32	16	8	4	2	1

4. Now, we look at the next power of 2 which is 2 to the 3rd which resolves to 8 and check to see if 8 can be subtracted from 11. It can, therefore we put a 1 in the 2 to the 3rd position. Then we subtract 11 - 8 = 3

$$215_{10} = \underline{11010111}_2$$

$$
\begin{array}{r}
215 \\
-128 \\
\hline
87 \\
-64 \\
\hline
23 \\
-16 \\
\hline
7 \\
-4 \\
\hline
3 \\
-2 \\
\hline
1 \\
-1 \\
\hline
0
\end{array}
$$

	1	1	0	1	0	1	1	1
~~256~~	128	64	32	16	8	4	2	1

$120_2 = \underline{1111000}_{10}$

$$\begin{array}{r} 120 \\ -\ 64 \\ \hline 56 \\ -\ 32 \\ \hline 24 \\ -\ 16 \\ \hline 8 \\ -\ 8 \\ \hline 0 \end{array}$$

1	1	1	1	0	0	0
64	32	16	8	4	2	1

```
   1  1
   1011      11
   1100      12
 + 1010      10
 --------   ----
  100001     33
32 16 8 4 2 1   Dec
     33
```

```
   1
 + 1
 ----
  10
 + 1
 ----
  11
 + 1
 ----
 100
```

```
 1  1  0  0
 1  0  1  1
 -  -  -  -
 8  4  2  1
```

10 1
101 — 5
111 — 7
110 — 6
+ 111 — 7

11001 — 25
16 8 4 2 1

16+8+1=25

~~11010~~
11001

$$\begin{array}{rrrr} & {}^{10} & {}^{10} & \\ \cancel{1} & \cancel{1} & 0 & 1 \\ - & 1 & 1 & 0 \\ \hline & 1 & 1 & 1 \end{array} = 7 \leftrightarrow 7$$

$$\begin{array}{r} 13 \\ -6 \\ \hline \end{array}$$

Dec

$$1\ 1\ 0\ 1$$

$$\frac{\quad}{8}\ \frac{\quad}{4}\ \frac{\quad}{2}\ \frac{\quad}{1}$$

Dec

$$
\begin{array}{rcl}
\cancel{1}\,\overset{\cancel{10}}{0}\,\overset{10}{0}\,1 & = & 9 \\
-\quad 1\,1\,0 & = & 6 \\
\hline
1\,1 & = & 3 \Leftrightarrow 3
\end{array}
$$

$\overline{1}\,\overline{0}\,\overline{0}\,\overline{1}$

$$\begin{array}{r} \cancel{1}\,\cancel{0}\,0\,\cancel{1}\,0\,1 \\ -\quad 1\,0\,1\,1 \\ \hline 1\,1\,0\,1\,0 \\ \hline 1\,0\,0\,1\,0\,1 \end{array}$$

$$\begin{array}{r} 37 \\ -\ 11 \\ \hline 26 \end{array}$$

$$\begin{array}{ccccc} 1 & 1 & 0 & 1 & 0 \\ 16 & 8 & 4 & 2 & 1 \end{array}$$

$$\underbrace{\qquad\qquad}_{26}$$

Numbering Systems and Comp...

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to the next position D therefore, 1 + F = 10.

Binary	Decimal	Hexadecimal
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F
10000	16	10
10001	17	11
10010	18	12

Conversion from binary to hexadecimal:

Binary Hexadecimal

Base 16

Digits 0-F

Dec	Bin	Hex
$\begin{array}{r} 9 \\ +\ 1 \\ \hline 10 \end{array}$	$\begin{array}{r} 1 \\ +\ 1 \\ \hline 10 \end{array}$	$\begin{array}{r} F \\ +\ 1 \\ \hline 10 \end{array}$

$$\begin{array}{r} FF \\ +\ \ 1 \\ \hline 100 \end{array} \qquad \begin{array}{r} 1F \\ +\ \ 1 \\ \hline 20 \end{array}$$

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to the next position D therefore, 1 + F = 10.

Binary	Decimal	Hexadecimal
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F
10000	16	10
10001	17	11
10010	18	12

Conversion from binary to hexadecimal:

Binary Hexadecimal

ASCII

8 bits = byte

Character

F				B			
1	1	1	1	1	0	1	1
8	4	2	1	8	4	2	1

15 Dec

F hex

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to the next position D therefore, 1 + F = 10.

Binary	Decimal	Hexadecimal
0	0	0
1	1	1
10	2	2
11	3	3
100	4	4
101	5	5
110	6	6
111	7	7
1000	8	8
1001	9	9
1010	10	A
1011	11	B
1100	12	C
1101	13	D
1110	14	E
1111	15	F
10000	16	10
10001	17	11
10010	18	12
10011	19	13

$$\begin{array}{r} 9 \\ +\ 1 \\ \hline A \\ +\ 1 \\ \hline B \\ +\ 1 \\ \hline C \\ +\ 1 \\ \hline D \\ +\ 1 \\ \hline E \end{array} \qquad \begin{array}{r} E \\ +\ 1 \\ \hline F \\ +\ 1 \\ \hline 10 \end{array}$$

Conversion from binary to hexadecimal:

Binary Hexadecimal